

## Curvature Estimation for Unstructured Triangulations of Surfaces

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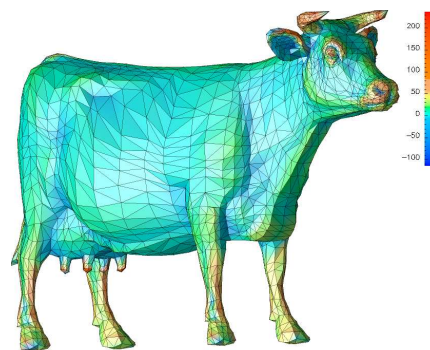
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Knowledge of the curvature of surfaces is important in a number of applications such as flow simulations, computer graphics and animations, and pattern matching. It is of particular importance to applications dealing with evolving surface geometry. Such applications usually do not have smooth analytical forms for the surfaces forming the model geometry. Instead they have to deal with discrete data consisting of points on the surface connected to form a unstructured triangulation. Hence, it is important to be able to reliably estimate local curvatures at points on discrete surfaces.

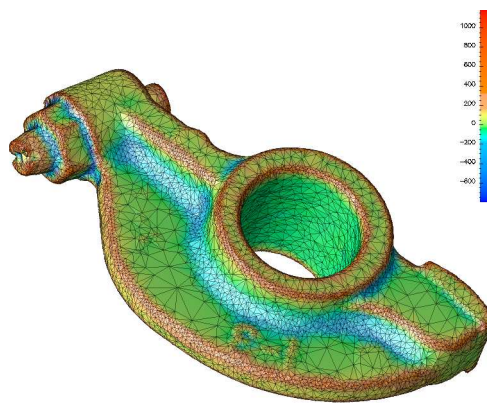
In this study, a comparative analysis and convergence study was done of several curvature estimation methods suggested in the literature and a new, improved method was proposed as an extension of one of those methods [2]. The new method robustly estimates normals, principal curvatures, mean curvatures and Gaussian curvatures at vertices of general unstructured triangulations. The method has been tested on complex meshes and has provided very good results.

It was found that the most reliable methods are those that fit a smooth surface to a set of nodes in a local neighborhood of each node and use its curvatures as the estimate of the curvatures for the discrete surface [1]. Since surface curvatures are based on second order derivatives, it is common to use quadratic polynomials as a local approximation to the surface. In each of these methods, a quadric of the form  $Z' = f(X', Y')$  is fitted to the nodes in a local coordinate system  $(X', Y', Z')$  whose origin is at the node under consideration and whose  $Z'$  axis is along an estimated surface normal at the node.

The simplest of the surface fitting methods performs a least squares fit of the quadric  $Z' = aX'^2 +$



(a)



(b)

*Mean curvature estimates for cow and rocker arm  
(mesh courtesy of Cyberware, Inc.)*

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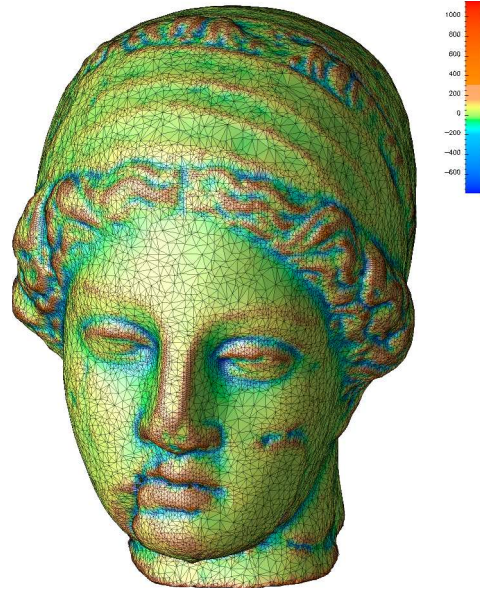
$bX'Y' + cY'^2$  to the 1-level or edge-connected neighbors of the node mapped to a local coordinate system. The Z-coordinate of the local coordinate system is along a particular normal that is approximated by averaging the normals of the triangles connected to the node.

A more robust version of the quadric fitting method includes linear terms in the quadric, i.e.,  $Z' = aX'^2 + bX'Y' + cY'^2 + dX' + eY'$ . This yields a new estimate for the tangent plane and normal at the node. The new normal is then used to calculate a new local coordinate frame in which to fit a new quadric. This iterative process is carried out until the local coordinate frame does not change much. The extended quadric fitting works well but requires more points than the simple quadric without which the system of equations becomes under-determined and does not have a unique solution. It is also possible to use a full quadric in which the addition of the constant term allows the surface to not pass through the node under consideration but that was not considered here.

The method devised by Garimella and Swartz fits an extended quadric to the 1-level neighborhood of the node whenever possible and includes the 2-level neighbors of the node (neighbors of the node neighbors) when the system is under-determined. The method also includes a technique for estimating curvature at vertices on surface boundaries where a full cycle of faces may not exist around the vertex. At such boundary nodes, existing neighbors are reflected to form ghost nodes that can then be used for fitting the surface.

In the study, convergence tests were performed by estimating the curvature at the central node of a shrinking hexagonal patch of triangles superimposed on cylinder. The hexagonal patch was shrunk in a self-similar way so that the quality of and relative sizes of the triangles did not change. It was found that the simple quadric fitting converged to the wrong solution while the extended quadric technique showed very good accuracy and convergence to the right solution. In other tests involving the triangulation of a sphere,

similar results were obtained with the addition that the extended quadric, extended patch method



*Mean curvature estimates for Igea artifact (mesh courtesy of Cyberware, Inc.)*

of Garimella and Swartz was needed to get an estimate at points in the mesh which did not have enough 1-level neighbors.

The modified curvature estimation method was tried on complex meshes and has shown to give very good results as shown in the figures.

## References

- [1] A. M. McIVOR AND R. J. VALKENBURG. A comparison of local surface geometry estimation methods. *Machine Vision and Applications*, 10:17–26, 1997.
- [2] S. PETITJEAN. A survey of methods for recovering quadrics in triangle meshes. *ACM Computing Surveys*, 34(2):211–262, June 2002.

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